PRECALCULUS NOTES 9.19.11 - 9.21.11

1.3 TEN BASIC FUNCTIONS

Linear Function $f(x) = mx + b$

- The domain and range of the function is the set of all real numbers
- The graph has a x-intercept of (-b/m , 0)
- The graph has a y-intercept of (0, b)
- The graph is increasing if m> 0
- The graph is decreasing if m<0
- The graph is constant if m = 0

Identity Function

- A special type of linear function
- Slope = 1
- Y-intercept at (0,0)
- The graph is always increasing

Squaring Function $f(x) = x^2$

- U-shaped curve
- Domain: set of all real numbers Range: set of all real positive numbers
- Function is even
- Graph has an intercept at (0,0)
- The graph is decreasing on the interval (- neg inf., 0) and increasing on the interval (0, pos. inf.)
- The graph is symmetric to the y-axis
- The graph as a relative minimum at (0,0)

Cubic Function $f(x) = x^3$

- The origin is called the point of inflection because the curve changes at this point
- Domain: set of all real numbers Range: set of all real numbers
- The function is odd
- The graph as an intercept at (0,0)
- The graph is increasing on the interval (negative infinity, positive infinity)
- The graph is symmetric with respect to the origin

Square root f(x) = square root of x

- Domain: all positive real numbers Range: set of all positive real numbers
- Graph has an intercept at (0,0)
- The graph is increasing on the interval (0, infinity)

Reciprocal $f(x) = 1/x$

- The shape of the curve is called a hyberbola
- Continuous on every point in its domain so its considered to be a continuous function
- Domain (neg. inf., 0) U (0, pos. inf.) Range (neg. inf., 0) U (0, pos. inf.)
- The function is odd
- The graph does not have any intercepts
- The graph is decreasing on the intervals (neg. inf., 0) and (0, pos. inf.)
- The graph is symmetric with respect to the origin.

Exponential $f(x) = e^x$

Natural logarithm $f(x) = \ln x$

Sine Function $f(x) = \sin x$

Cosine Function $f(x) = \cos x$

Absolute Value $f(x) = \text{abs}(x)$

Function	Domain	Range	VA	HA	Discont	Interval Inc/Dec	Symmetry
$f(x) = x$							
$f(x) = x^2$							
$f(x) = x^3$							
f(x) = square root of x							
$f(x) = e^x$							
$f(x) = \ln x$							
$f(x) = \cos x$							
$f(x) = \sin x$							
$f(x) = \text{abs}(x)$							

Identifying Piecewise – Defined Functions

1. Which function has the following piecewise function over different intervals of its domain?

$$f(x) = \begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$$

2. Sketch a graph of f(x)

$$f(x) = \begin{cases} 2x + 3, & x \leq 1 \\ -x + 4, & x > 1 \end{cases}$$

3. Graph $f(x) = (x - 3)^2$

   - What intervals is the function increasing?

   - What intervals is the function decreasing?

   - Is the function even, odd, or neither?

   - What are the extrema?

   - How does the graph relate to $f(x) = x^2$

In class, review and discuss pg. 105 (2-20 even)
Homework: pg: 105 (21-26, skip 23, 31-34)

1.4 BUILDING FUNCTIONS FROM FUNCTIONS

$(f + g)(x) = f(x) + g(x)$

$(f - g)(x) = f(x) - g(x)$

$(f\ g)(x) = f(x)g(x)$

$(f / g)(x) = f(x)/g(x)$

Defining New Functions Algebraically

1.) $f(x) = 2x - 1$; $g(x) = x^2$

- Find the domain of each function

- Where do the domains overlap?

- Find (f + g), (f - g), (f g), & (f /g) and state the domain of each.

2.) f(x) = square root x; g(x) = sinx

- Find the domain of each function

- Where do the domains overlap?

- Find (f + g), (f - g), (f g), & (f /g) and state the domain of each.

Composite Functions

$(f \circ g)(x) = f(g(x))$

x must be in the domain of g (x) and g(x) must be in the domain of f(x)

Examples

1.) Given f(x) = 2x + 1 $g(x) = 4 - x^2$

Find the following:

a.) $(f \circ g)(x)$ b.) $(g \circ f)(x)$ c.) $(g \circ f)(-2)$

2.) Given $f(x) = x^2$ g(x) = x-1

Find the following:

a.) $(f \circ g)(x)$ b.) $(g \circ f)(x)$ c.) $(g \circ f)(-2)$

3.) Given f(x) = square root (x+4) $g(x) = x^2$

Find the following:

b.) $(f \circ g)(x)$ b.) $(g \circ f)(x)$

c.) What is the domain of f(x)?

d.) What is the domain of g(x)?

e.) What is the domain of $(f \circ g)(x)$?

f.) What is the domain of $(g \circ f)(x)$?

Decomposing a Composite Function

Write the function $h(x) = 1/(x-2)^2$ as a composition of two functions.

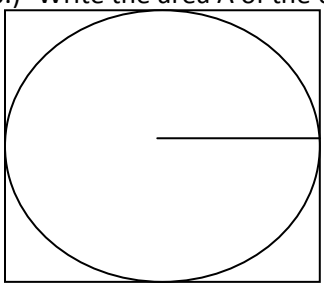
…so h(x) = f (g(x)) What are f(x) and g(x)?

Word Problem Using Composition Functions

A square concrete foundation is prepared as a base for a cylindrical tank.

a.) If the edge of the square foundation is x and the radius of the circle is r, write the radius of the tank as a function of the length of the x of the sides of the square.

b.) Write the area A of the circular base of the tank as a function of r.



c.) Find and interpret $(A \circ r)(x)$.