Systems of Linear Equations
Monetary Systems Overload

ACTIVITY
1.2

SUGGESTED LEARNING STRATEGIES: Shared Reading, Close Reading, Interactive Word Wall

My Notes

Have you ever noticed that when an item is popular and many people want to buy it, the price goes up? Have you ever noticed that items that no one wants are marked down to a lower price?

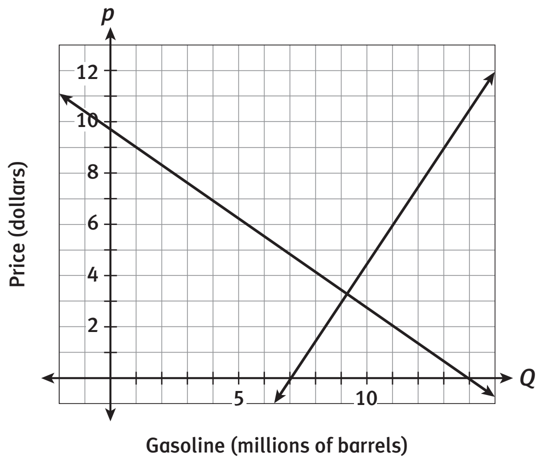
The change in an item's price and the quantity available to buy are the basis of the concept of *supply and demand* in economics. Demand refers to the quantity that people are willing to buy at a particular price. Supply refers to the quantity that the manufacturer is willing to produce at a particular price. The final price that the customer sees is a result of both supply and demand.

Suppose that during a six-month time period, the supply and demand for gasoline has been tracked and approximated by these functions, where Q represents millions of barrels of gasoline and P represents price per gallon in dollars.

- Demand function: $P = -0.7Q + 9.7$
- Supply function: $P = 1.5Q - 10.4$

To find the best balance between market price and quantity of gasoline supplied, find a **solution of a system of two linear equations**. The demand and supply functions for gasoline are graphed below.

MATH TERMS
A point, or set of points, is a **solution of a system of equations** in two variables when the coordinates of the points make both equations true.



1. Find an approximation of the coordinates of the intersection of the supply and demand functions. Explain what the point represents.

SUGGESTED LEARNING STRATEGIES: Create Representations, Vocabulary Organizer

My Notes

TECHNOLOGY TIP

You can use a graphing calculator and its TRACE function to solve systems of equations in two variables.

MATH TERMS

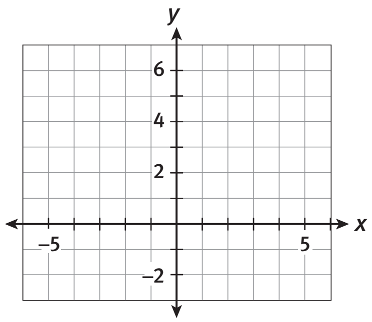
Systems of linear equations are classified by the number of solutions.

- Systems with no solution are **inconsistent**.
- Systems with one or many solutions are **consistent**.
- A system with exactly one solution is **independent**.
- A system with infinite solutions is **dependent**.

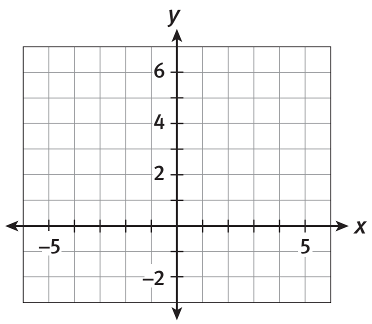
2. What problem(s) can arise when solving a system of equations by graphing?

3. Graph each system. Determine the number of solutions.

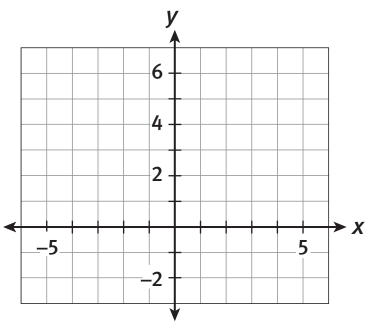
a. $\begin{cases} y = x + 1 \\ y = -x + 4 \end{cases}$



b. $\begin{cases} y = 5 + 2x \\ y = 2x \end{cases}$



c. $\begin{cases} y = 2x + 1 \\ 2y = 2 + 4x \end{cases}$



4. Graphing two linear equations illustrates the relationships of the lines. Classify the systems in Item 3 as **consistent and independent, consistent and dependent,** or **inconsistent**.

SUGGESTED LEARNING STRATEGIES: Note Taking, Look for a Pattern, Think/Pair/Share

My Notes

Investors try to control the level of risk in their portfolios by diversifying their investments. You can solve some investment problems by writing and solving systems of equations. One algebraic method for solving a system of linear equations is called **substitution**.

MATH TERMS

In the **substitution method,** you solve one equation for one variable in terms of another. Then substitute that expression into the other equation to form a new equation with only one variable. Solve that equation. Substitute the solution into one of the two original equations to find the other variable.

EXAMPLE 1

During one year, Sara invested \$5000 into two separate funds, one earning 2% and another earning 5% annual interest. The interest Sara earned was \$205. How much money did she invest in each fund?

Step 1: *Let x = money in the first fund and y = money in the second fund.*

Write one equation to represent the amount of money invested.
Write another equation to represent the interest earned.

$x + y = 5000$ — The money invested is \$5000.
$0.02x + 0.05y = 205$ — The interest earned is \$205.

Step 2: *Use substitution to solve this system.*

$x + y = 5000$ — Solve the first equation for y.
$y = 5000 - x$
$0.02x + 0.05(5000 - x) = 205$ — Substitute for y in the second equation.
$0.02x + 250 - 0.05x = 205$ — Solve for x.
$-0.03x = -45$
$x = 1500$

Step 3: *Substitute the value of x into one of the original equations to find y.*

$x + y = 5000$
$1500 + y = 5000$ — Substitute 1500 for x.
$y = 3500$

Solution: Sara invested \$1500 in the first fund and \$3500 in the second fund.

MATH TIP

Check your answer by substituting the solution (1500, 3500) into the second original equation.
$0.02x + 0.05y = 205$

TRY THESE A

Write your answers on notebook paper. Show your work.

Solve each system of equations, using substitution.

a. $\begin{cases} x = 25 - 3y \\ 4x + 5y = 9 \end{cases}$ **b.** $\begin{cases} x + 2y = 14 \\ 2y = x - 10 \end{cases}$ **c.** $\begin{cases} y - x = 4 \\ 3x + y = 16 \end{cases}$

5. When using substitution, how do you decide which variable to isolate and which equation to solve? Explain.

SUGGESTED LEARNING STRATEGIES: Note Taking

My Notes

MATH TERMS

The *elimination method* is also called the *addition-elimination* or the *linear combination method* for solving a system of linear equations.

MATH TERMS

In the **elimination method,** you *eliminate* one variable. Multiply each equation by a number so that the terms for one variable combine to 0 when the equations are added. Then use substitution with that value of the variable to find the value of the other variable. The ordered pair is the *solution* of the system.

Another algebraic method for solving systems of linear equations is the **elimination method**.

EXAMPLE 2

A stack of 20 coins contains only nickels and quarters and has a total value of $4. How many of each coin are in the stack?

Step 1: *Let n = number of nickels and q = number of quarters.*

Write one equation to represent the number of coins in the stack. Write another equation to represent the total value.

$n + q = 20$	The number of coins is 20.
$5n + 25q = 400$	The total value is 400 cents.

Step 2: *To solve this system of equations, first eliminate the n variable.*

$-5(n + q) = -5(20)$	Multiply the first equation by -5.
$5n + 25q = 400$	
$-5n - 5q = -100$	
$\underline{5n + 25q = 400}$	Add the two equations to eliminate n.
$20q = 300$	Solve for q.
$q = 15$	

Step 3: *Find the value of the eliminated variable n by using the original first equation.*

$n + q = 20$	
$n + 15 = 20$	Substitute 15 for q.
$n = 5$	

Step 4: *Check your answers by substituting into the original second equation.*

$5n + 25q = 400$	
$5(5) + 25(15) \ ? \ 400$	Substitute 5 for n and 15 for q.
$25 + 375 \ ? \ 400$	
$400 = 400$	Check.

Solution: There are 5 nickels and 15 quarters in the stack of coins.

SUGGESTED LEARNING STRATEGIES: Close Reading, Vocabulary Organizer

My Notes

TRY THESE **B**

Solve each system of equations, using elimination. Write your answers in the My Notes space. Show your work.

a. $\begin{cases} -2x - 3y = 5 \\ -5x + 3y = -40 \end{cases}$ **b.** $\begin{cases} 5x + 6y = -14 \\ x - 2y = 10 \end{cases}$ **c.** $\begin{cases} -3x + 3y = 21 \\ -x - 5y = -17 \end{cases}$

Sometimes a situation has more than two pieces of information. For these more complex problems, you may need to solve equations that contain three variables.

In Bisbee, Arizona, an old mining town, you can buy souvenir nuggets of gold, silver, and bronze. For $20, you can buy any of these mixtures of nuggets: 14 gold, 20 silver, 24 bronze; or, 20 gold, 15 silver, 19 bronze; or, 30 gold, 5 silver, 13 bronze. What is the monetary value of each souvenir nugget?

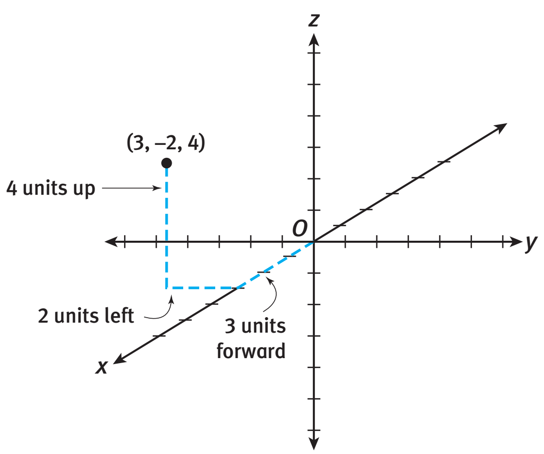
The problem above represents a system of linear equations in three variables. The system can be represented with these equations.

$$\begin{cases} 14g + 20s + 24b = 20 \\ 20g + 15s + 19b = 20 \\ 30g + 5s + 13b = 20 \end{cases}$$

Although it is possible to solve systems of equations in three variables with the substitution method, it can be difficult. It can also be very challenging to solve this kind of system by graphing.

Just as the ordered pair (x, y) is a solution of a system in two variables, the **ordered triple** (x, y, z) is a solution of a system in three variables. Ordered triples are graphed in three-dimensional coordinate space.

The point $(3, -2, 4)$ is graphed below.



MATH TIP

An ordered pair can also be the solution of a single equation in two variables. Likewise, an ordered triple can also be the solution of a single equation in three variables.

ACTIVITY 1.2 *continued*

Systems of Linear Equations

Monetary Systems Overload

SUGGESTED LEARNING STRATEGIES: Note Taking

My Notes

MATH TIP

You can use either elimination or substitution to solve a system of three equations in three variables.

Use elimination if the terms easily add to 0. Use substitution if one equation has only one variable on one side, such as $x = 2y + z$.

The elimination method is usually the easiest method for solving systems of equations in three variables.

EXAMPLE 3

Solve this system, using elimination. $\begin{cases} 2x + 7y + z = -53 \\ -2x + 3y + z = -13 \\ 6x + 3y + z = -45 \end{cases}$

Step 1: *Add the first and second equations to eliminate x.*

$$\begin{aligned} 2x + 7y + z &= -53 \\ \underline{-2x + 3y + z} &\underline{= -13} \\ 10y + 2z &= -66 \end{aligned}$$

Step 2: *Use the second and third equations to eliminate x again. Multiply the second equation by 3 so that the x-terms add to zero.*

$$\begin{aligned} 3(-2x + 3y + z) = 3(-13) &\longrightarrow & -6x + 9y + 3z &= -39 \\ 6x + 3y + z = \ -45 &\longrightarrow & \underline{6x + 3y + \ z} &\underline{= -45} \\ & & 12y + 4z &= -84 \end{aligned}$$

Step 3: *Use the two equations you found to write a new system in two variables. Multiply the second equation by −2 so that the z-terms add to zero.*

New system	Multiply the first equation by −2.	Add the equations to eliminate z.

$$\begin{aligned} 10y + 2z = -66 &\longrightarrow -2(10y + 2z) = -2(-66) \longrightarrow & -20y - 4z &= 132 \\ 12y + 4z = -84 &\longrightarrow \quad 12y + 4z \ = -84 \qquad\quad \longrightarrow & \underline{12y + 4z} &\underline{= -84} \\ & & -8y &= 48 \\ & \text{Solve for } y. & y &= -6 \end{aligned}$$

Step 4: *Substitute the y-value into one of the two new equations to find z.*

$$\begin{aligned} 10y + 2z &= -66 \\ 10(-6) + 2z &= -66 \\ -60 + 2z &= -66 \\ 2z &= -6 \\ z &= -3 \end{aligned}$$

Step 5: *Substitute the y- and z-values into one of the original equations to find x.*

$$\begin{aligned} -2x + \quad 3y \ + z \quad &= -13 \\ -2x + 3(-6) + -3 &= -13 \\ -2x \qquad\quad - \ 21 &= -13 \\ -2x \qquad\qquad\quad &\ \ \ 8 \\ x \qquad\qquad\quad &= \ -4 \end{aligned}$$

Solution: The solution of the system is (−4, −6, −3).

MATH TIP

As a final step, check your ordered triple solution in another original equation to be sure that your solution is correct.

SUGGESTED LEARNING STRATEGIES: Note Taking, Vocabulary Organizer, Look for a Pattern

My Notes

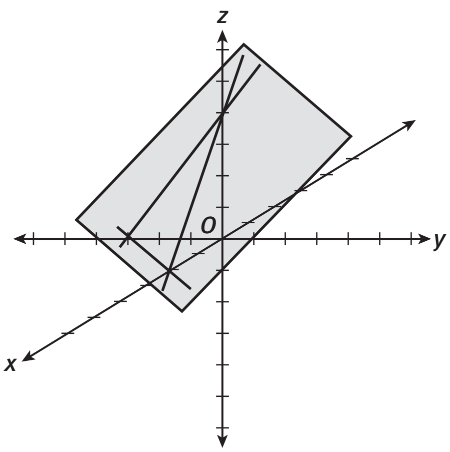
TRY THESE C

Solve each system of equations using elimination. Write your answers on notebook paper. Show your work.

a. $\begin{cases} x + y + z = 6 \\ 2x - y + 3z = 9 \\ -x + 2y + 2z = 9 \end{cases}$

b. $\begin{cases} x - 3y + 2z = -8 \\ 2x + 3y + z = 17 \\ 5x - 2y + 3z = 5 \end{cases}$

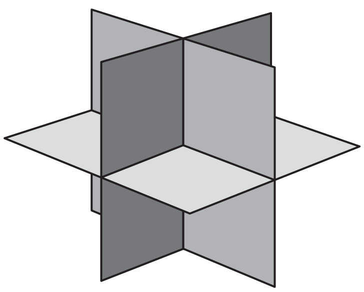
The graph of an equation in three variables is a plane in coordinate space.



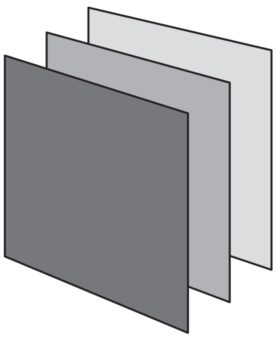
You can represent a system of equations in three variables as three planes in the *same* coordinate space. The graph of the solutions of a system in three variables is the intersection of the three planes. Only those points that form the intersection of *all three* planes represent the solution.

6. Classify each system as **consistent and independent, consistent and dependent,** or **inconsistent**.

a.



b.



MATH TIP

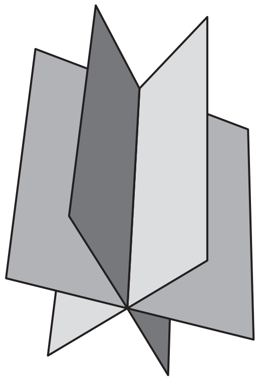
Systems of linear equations in three variables can be classified in the same way as systems in two variables. See Math Terms on page 14.

SUGGESTED LEARNING STRATEGIES: Look for a Pattern

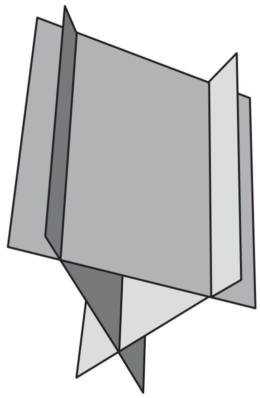
My Notes

6. *(continued)*

c.



d.



CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. Solve the system by graphing.

$$\begin{cases} 2x + 9 = y \\ y = -4x - 3 \end{cases}$$

2. Solve the system, using substitution.

$$\begin{cases} 4y + 19 = x \\ 3y - x = -13 \end{cases}$$

3. Solve the system, using elimination.

$$\begin{cases} 3x + 2y = 17 \\ 4x - 2y = 4 \end{cases}$$

4. Solve the system, using elimination.

$$\begin{cases} 3x - 2y + 7z = 13 \\ x + 8y - 6z = -47 \\ 7x - 9y - 9z = -3 \end{cases}$$

5. **MATHEMATICAL REFLECTION** Which solution method for solving systems of equations do you find easiest to use? Which method do you find most difficult to use? Explain why.