

Introduction to Functions

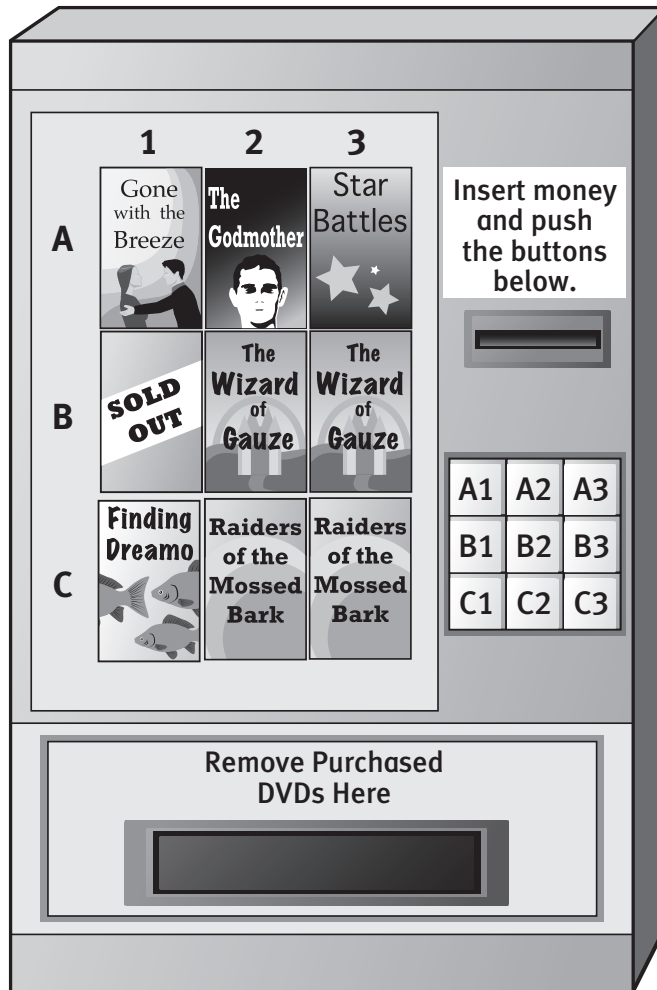
Vending Machines

SUGGESTED LEARNING STRATEGIES: Visualization

My Notes

Use this machine to answer the questions on the next page.

DVD Vending Machine



My Notes

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge

1. Suppose you inserted your money and pressed A1. What item would you receive?
2. Suppose you inserted your money and pressed C2. What item would you receive?
3. Suppose you inserted your money and pressed B3. What item would you receive?
4. If the machine were filled properly, what would happen if you pressed any of those same buttons again?

Each time you press a button, an **input**, you may receive a DVD, an **output**.

5. In the DVD vending machine situation, does every input have an output? Explain your response.
6. Each combination of input and output can be expressed as a **mapping** written *input* \rightarrow *output*. For example, B2 \rightarrow Wizard of Gauze]
 - a. Write as mappings each of the possible combinations of buttons pushed and DVDs received in the vending machine.

MATH TERMS

A **mapping** is a visual representation of a relation in which an arrow associates each input with its output.

CONNECT TO AP

When conducting observational studies in AP Statistics, the data collected are not always numerical. For example, a study might compare the fruit-juice flavor preferred by male students compared with the flavor preferred by female students.

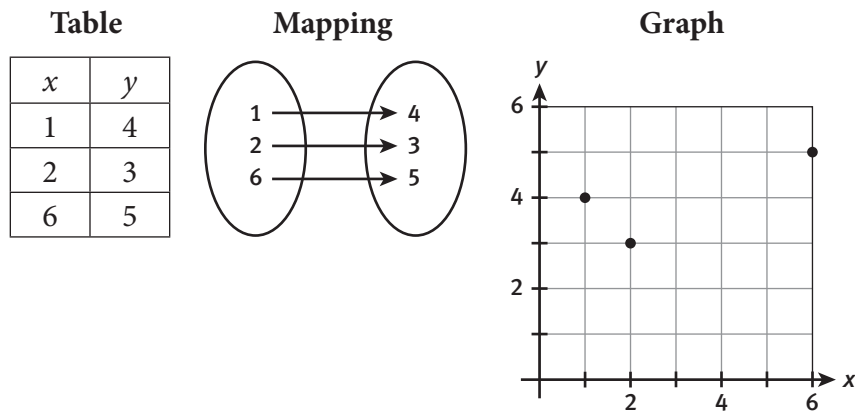
SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Note Taking, Create Representations

- b. Mappings relating values from one set of numbers to another set of numbers can be written as **ordered pairs**. Write the following numerical mappings as ordered pairs.

Input	→	Output	Ordered Pairs
1	→	-2	(1, -2)
2	→	1	
3	→	4	
4	→	7	

A **relation** is a set of ordered pairs. The list of ordered pairs that you wrote in Item 6(b) is a relation.

Relations can have a variety of representations. Consider the relation $\{(1, 4), (2, 3), (6, 5)\}$, shown here as a set of ordered pairs. This relation can also be represented in these ways.



7. You represented the vending machine situation using mappings in Item 6. Other representations can also be used to illustrate how the inputs and outputs of the vending machine are related.

- a. Create a table to illustrate how the inputs and outputs of the vending machine are related.
- b. In representing the vending machine inputs and outputs, what decisions would need to be made to create the graph?

My Notes

MATH TERMS

An **ordered pair** shows the relationship between two elements, written in a specific order using parentheses notation and a comma separating the two values.

MATH TERMS

relation

My Notes

ACADEMIC VOCABULARY

function

SUGGESTED LEARNING STRATEGIES: Group Presentation, Create Representations

A **function** is a relation in which each input is paired with exactly one output.

8. Compare and contrast the DVD Vending Machine with a function.

9. Suppose when pressing button C1 button on the vending machine both “Finding Dreamo” and “Raiders of the Mossed Bark” come out. How does this vending machine resemble or not resemble a function?

10. Imagine a machine where you input an age and the machine gives you the name of anyone who is that age. Compare and contrast this machine with a function. Explain by using examples and create a representation of the situation.

11. Create an example of a situation (math or real-life) that behaves like a function and another that does not behave like a function. Explain why you chose each example to fit the category.
 - a. Behaves like a function:

 - b. Does not behave like a function:

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Think/Pair/Share

12. Identify whether each list of ordered pairs represents a function. Explain your answers.

a. $\{(5, 4), (6, 3), (7, 2)\}$

b. $\{(4, 5), (4, 3), (5, 2)\}$

c. $\{(5, 4), (6, 4), (7, 4)\}$

13. Using positive integers, write two relations as a list of ordered pairs below, one that is a function and one that is not a function.

Function:

Not a function:

The set of all inputs for a function is known as the **domain** of the function. The set of all outputs for a function is known as the **range** of the function.

14. Consider a vending machine where inserting 25 cents dispenses one pencil, inserting 50 cents dispenses 2 pencils, and so forth up to and including all 10 pencils in the vending machine.

a. What is the domain in this situation?

b. What is the range in this situation?

My Notes

ACADEMIC VOCABULARY

domain
range

WRITING MATH

The **domain** and **range** of a function can be written using set notation.

For example for the function $\{(1,2), (3,4), (5,6)\}$ the domain is $\{1,3,5\}$ and the range is $\{2,4,6\}$.

My Notes

SUGGESTED LEARNING STRATEGIES: Quickwrite

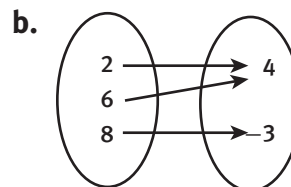
15. For each function below, identify the domain and range.

a.

input	output
7	6
3	-2
5	1

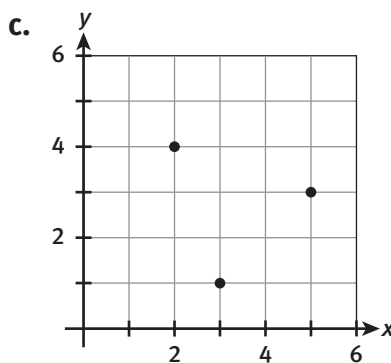
Domain:

Range:



Domain:

Range:



Domain:

Range:

d. $\{(-7, 0)(9, -3)(-6, 2.5)\}$

Domain:

Range:

MATH TERMS

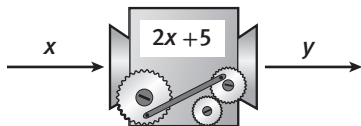
A **finite** set has a fixed countable number of elements.

An **infinite** set has an unlimited number of elements.

16. Each of the functions that you have seen has a **finite** number of ordered pairs. There are functions that have an **infinite** number of ordered pairs. Describe any difficulties that may exist trying to represent a function with an infinite number of ordered pairs using the four representations of functions that have been described thus far.

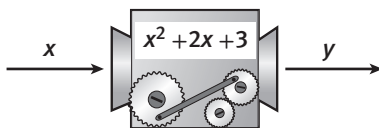
SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge

17. Sometimes, machine diagrams are used to represent functions. In the function machine below, the inputs are labeled x and the outputs are labeled y . The function is represented by the expression $2x + 5$.



- If $x = 7$ is used as an input, what is the output?
- If $x = -2$ is used as an input, what is the output?
- If $x = \frac{1}{2}$ is used as an input, what is the output?
- Is there any limit to the number of input values that can be used with this expression? Explain.

Consider the function machine below.



18. Use the diagram to find the (input, output) ordered pairs for the following values.
- $x = -5$
 - $x = \frac{3}{5}$
 - $x = -10$

My Notes

My Notes

SUGGESTED LEARNING STRATEGIES: Create Representations, Activating Prior Knowledge, Group Discussion

19. Make a function machine for the expression $10 - 5x$. Use it to find ordered pairs for $x = 3$, $x = -6$, $x = 0.25$, and $x = \frac{3}{4}$.

Creating a function machine can be time consuming and awkward. The function represented by the diagram in Item 17 can also be written algebraically as the equation $y = 2x + 5$.

20. Evaluate each function for $x = -2$, $x = 5$, $x = \frac{2}{3}$, and $x = 0.75$. For each x -value, find the corresponding y -value. Place the results in a table.

a. $y = 9 - 4x$

b. $y = \frac{1}{x}$

When referring to the functions in Item 20, it can be confusing to distinguish among them since each begins with “ $y =$.” Function notation can be used to help distinguish among different functions.

For instance, the function $y = 9 - 4x$ in Item 20(a) can be written:

$$f(x) = 9 - 4x$$

This is read as “ f of x ” and $f(x)$ is equivalent to y .

“ f ” is the name of the function.

x is the input variable.

MATH TIP

It is important to recognize that $f(x)$ does not mean f multiplied by x .

SUGGESTED LEARNING STRATEGIES: Create Representations

- 21.** To distinguish among different functions, it is possible to use different names. Use the name h to write the function from Item 20b using function notation.

Function notation is useful for evaluating functions for multiple input values. To evaluate $f(x) = 9 - 4x$ for $x = 2$, you substitute 2 for the variable x and write $f(2) = 9 - 4(2)$. Simplifying the expression yields $f(2) = 1$.

- 22.** Use function notation to evaluate $f(x)$ shown above at $x = 5$, $x = -3$, and $x = 0.5$.
- 23.** Use the values for x and $f(x)$ from Item 22. Display the values using each representation.

a. list of ordered pairs

b. table of values

c. mapping

d. graph

- 24.** Evaluate each function for $x = -5$, $x = \frac{4}{3}$.

a. $f(x) = 2x - 7$

b. $g(x) = 6x - x^2$

c. $h(x) = \frac{2}{x^2}$

My Notes

MATH TIP

Notice that $f(x) = y$. For a **domain** value x , the associated **range** value is $f(x)$.

My Notes

A function whose domain is the set of positive consecutive integers forms a *sequence*. The terms of the sequence are the range values of the function. For the sequence $\{4, 7, 10, 13\}$, $f(1) = 4$, $f(2) = 7$, $f(3) = 10$, and $f(4) = 13$.

25. Consider the sequence $\{-4, -2, 0, 2, 4, 6, 8\}$.

- a. What is $f(3)$?
- b. What is $f(6)$?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper.

Show your work.

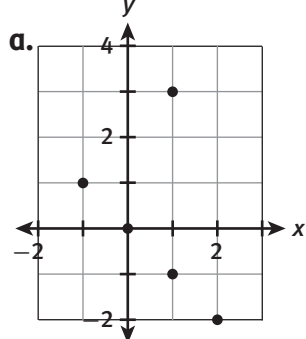
1. The set $\{(3,5), (-1,2), (2,2), (0,-1)\}$ represents a function. Identify the domain and range of the function. Then display the function using each representation.

a. a table

b. a mapping

c. a graph

2. Explain why each of the following is *not* a function.



b.

x	y
12	-8
17	3
-4	9
17	-5

3. Evaluate the functions for each domain value indicated.

a. $p(x) = 3x + 14$

$x = -5, 0, 4$

b. $h(t) = t^2 - 5t$

$t = -2, 0, 5, 7$

4. Consider the sequence $\{-7, -3, 1, 5, 9\}$.

a. What is $f(2)$?

b. What is $f(5)$?

5. MATHEMATICAL REFLECTION Which representation of a function do you feel is most useful? Why? Which one do you feel is least useful? Why?